Amplitudes from M2 and M5 branes

Yu-tin Huang
Arkani-Hamed, Chen, Dennen, Gang, Koh, Lee, Lipstein, Rozali, Siegel, Xie

UCLA/IAS

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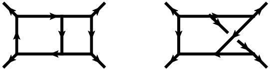
Why amplitudes?

Amplitudes can be viewed as auxiliary object at the bottom of perturbative tower:

- Begin with the S-matrix: $\langle g^- g^- g^+ g^+ \rangle$



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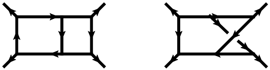
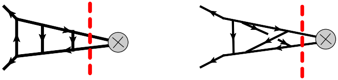


- Controls the branch cut structure of form factors: $\langle g^- g^+ | \mathcal{O}(k) | 0 \rangle$



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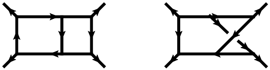

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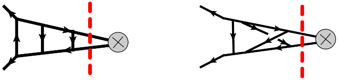
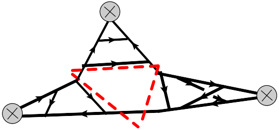


- Controls the branch cut structure of correlation functions: $\langle \mathcal{O}(1) \mathcal{O}(2) \mathcal{O}(3) \rangle$



The return of analytic S-matrix

Analytic approach: understanding perturbative physics without relying on an underlying Lagrangian

- Casts old concepts in new light:

$$c_2 \to \beta \text{ function}, \quad c_3 = 0 \to \mathrm{N} = 4 \text{ SYM}, \mathrm{N} = 8 \text{ SG}, \mathrm{N} = 2?,$$

- Reveals new symmetries or dualities:
  Dual superconformal symmetry (Yangian) for planar N=4 SYM,
  Dual superconformal symmetry (Yangian) for planar ABJM.
  Not understood in the string side
  UV finiteness of N=4 SUGRA (3Loops) Bern, Davies ,Dennen, me
  Is there a new symmetry?

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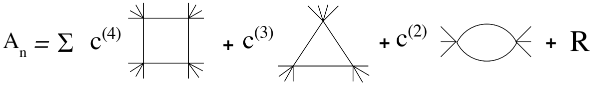

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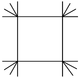
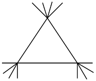
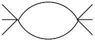
$A_n = \sum\ c^{(4)}$  $+\ c^{(3)}$  $+\ c^{(2)}$  $+\ R$

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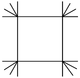
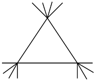
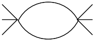
$A_n = \sum\ c^{(4)}$  $+\ c^{(3)}$  $+\ c^{(2)}$  $+\ R$

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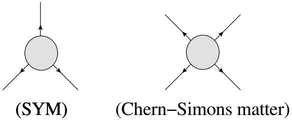

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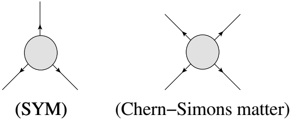

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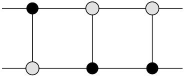
- X = Planar Integrand:



Equivalent to Dual superconformal symmetry

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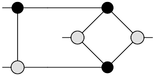
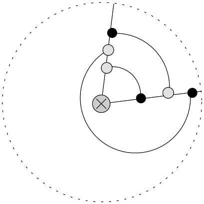


- X = Form factor Integrand: Arkani-Hamed, Xie, me



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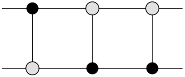

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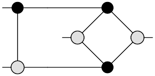

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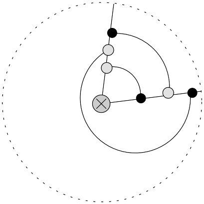

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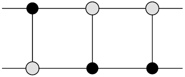

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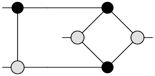

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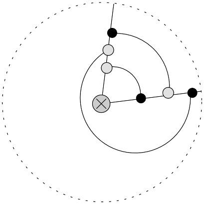

Why now?

$$\lambda^{A}{}_{a}$$

A ← Lorentz

a ← little

$$P^2 = 0 \rightarrow P^{AB} = \lambda^{Aa}\lambda^{B}{}_{a}, \quad \mathrm{Lorentz(A)} \times \mathrm{little(a)} - \mathrm{littlegroup} = \mathrm{D} - 1$$

$$D = 3, 4, 6$$

$$\mathrm{OSp(N|4)} \ \ (\mathrm{M2}), \quad \mathrm{SU(2,2|N)} \ \ (\mathrm{D3}), \quad \mathrm{OSp^*(8|N)} \ \ (\mathrm{M5})$$

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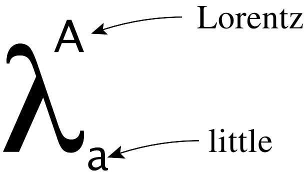

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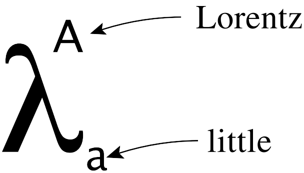

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All the superconformal generators can be written as acting on λ space.

- Determined the three-point S-matrix via symmetries.
- The result, (wether it exists or not), constrains the four-point.
- Once a non-trivial S-matrix element is found, cut constructible for the next order gives the gauge algebra.

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Example: Yang-Mills

- Super Poincare uniquely fix $A_3 = \frac{\delta^4(P)\delta^8(Q)}{\langle 12\rangle\langle 23\rangle\langle 31\rangle} f^{123}$
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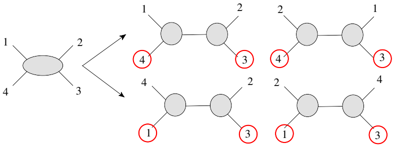
$$A_4|_{s\to 0} \sim \frac{A_3 \times A_3}{s}$$

- Cut constructibility:



$\rightarrow f^{[12a}f_a{}^{4]3}$

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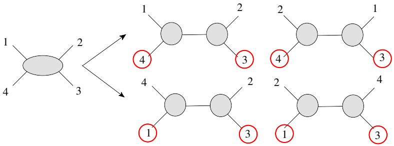

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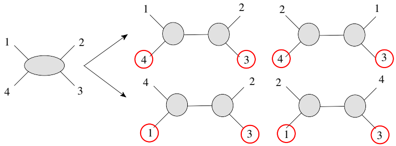

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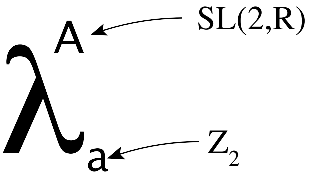
Outline

Three-dimensions

D=3



$\mathcal{N}=8$

Consder $\mathcal{N}=8$ $OSp(8|4)$
The supercofnormal generators (λ^α,η^I):

$$
\begin{aligned}
P^{\alpha\beta} &= \lambda^\alpha\lambda^\beta \quad (3)\\
Q^\alpha_I &= \frac{1}{\sqrt{2}}(\lambda^\alpha\frac{\partial}{\partial\eta^I}+\lambda^\alpha\eta_I) \quad (16)\\
M^\alpha{}_\beta &= \lambda^\alpha\frac{\partial}{\partial\lambda^\beta}-\delta^\alpha{}_\beta\lambda^\gamma\frac{\partial}{\partial\lambda^\gamma} \quad (3)\\
D &= \frac{1}{2}(\lambda^\alpha\frac{\partial}{\partial\lambda^\alpha}+1) \quad (1)\\
R^I{}_J &= \eta^I\eta_J-\frac{1}{2}\delta^I_J \quad (28)\\
S^I_\alpha &= \frac{1}{\sqrt{2}}(\eta^I\frac{\partial}{\partial\lambda^\alpha}+\frac{\partial}{\partial\lambda^\alpha}\frac{\partial}{\partial\eta_I}) \quad (16)\\
K_{\alpha\beta} &= \frac{\partial}{\partial\lambda^\alpha}\frac{\partial}{\partial\lambda^\beta} \quad (3).
\end{aligned}
$$

$\mathcal{N} = 8$

- D $A_n = \sum_i^n \frac{1}{2}(\lambda_i^\alpha \frac{\partial}{\partial \lambda_i^\alpha} + 1)$ A_n=0: only even S-matrix elements are non-trivial.
- $OSp(8|4)$: $A_4 = \delta^3(P)\delta^8(Q)/\langle ij\rangle\langle kl\rangle\langle mn\rangle$
- Vanishing factorization $A_4 = \delta^3(P)\delta^8(Q)/\langle 12\rangle\langle 23\rangle\langle 31\rangle$ f^{abcd} totally antisymmetric!
- Cut constructabilty requires $f^{[abcd}f_d{}^{e]fg} = 0$

Symmetry and factorization properties recover the content of BLG.

Reduction to $\mathcal{N} = 6$, ABJM.

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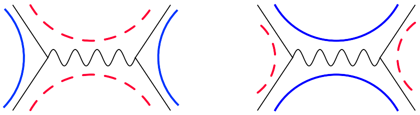
Intro on ABJM

Field content:
U(N)$_k\times$U(N)$_{-k}$ gauge fields $(\mathrm{A}^{\mu}_{ab}, \bar{\mathrm{A}}^{\mu ab})$,
SU(4) bi-fundamental matter $(\phi^{I}_{a}{}^{b}, \psi^{I}_{a}{}^{b}, \bar{\phi}_{I}{}^{a}{}_{b}, \bar{\psi}_{I}{}^{a}{}_{b})$

$$\mathcal{L} = \mathcal{L}_{C\ S} + \mathcal{L}_{\phi,Kin} + \mathcal{L}_{\psi,Kin} + \mathcal{L}_{4\phi^2\psi^2} + \mathcal{L}_{6\phi^6}$$

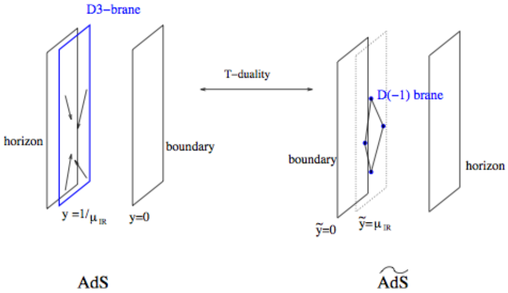


Parity invariance $k \leftrightarrow -k$, $A_n \sim k^{n/2-1}$:
Cyclic invariant by one site for n/2 $\in$ even
Cyclic invariant for n/2 $\in$ odd
$\rightarrow$ Only receives contribution from even loops

Strong Coupling

For $N, k \to \infty$ $\lambda = N/k$ dual to IIA string in $AdS_4 \times CP_3$

Flash back to N=4 SYM/ $AdS_5 \times S_5$ Beisert, Ricci, Tseytlin, Wolf, Maldacena,Berkovits



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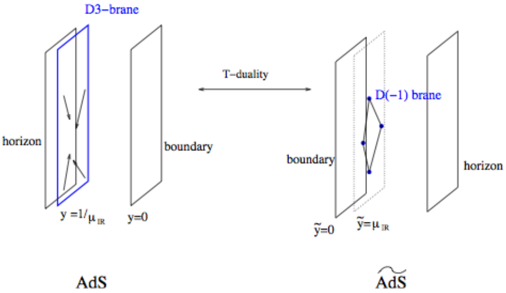

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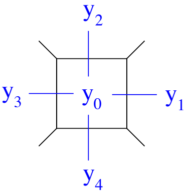
Weak Coupling (N=4SYM)

Dual conformal symmetry: Drummond, Henn, Smirnov, Sokatchev

$$A_4^{1\,\text{loop}} = st\, A_4^{\text{tree}}$$



$$I_4 = \int dy_0^4 \frac{1}{y_{01}^2 y_{02}^2 y_{03}^2 y_{04}^2}, \qquad I_C\,[\text{st}\, I_4] = \text{st}\, I_4$$

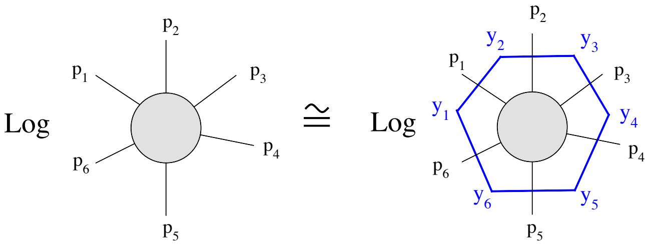
Dual superconformal symmetry:

$$I_C\left[A_n^{\text{tree}}\right] = \left(\prod_{i=1}^{n} y_i^2\right) A_n^{\text{tree}}$$

A symmetry for all loop planar integrand Drummond, Henn, Korchemsky, Sokatchev, Bern, me

Weak Coupling (N=4SYM)

Amplitude/Wilson-Loop Duality: Drummond, Henn, Korchemsky, Sokatchev



ABJM

Is there a similar story for ABJM?

Adam, Dekel, Oz; Grassi, Sorokin, Wulff "Fermionic T-duality cannot be implemented for AdS backgrounds of OSp cosets"

$\rightarrow$ There will be no dual superconformal symmetry

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However, Lipstein, me

$$I_C\left[A_4^{\rm tree}\right] = \left(\prod_{i=1}^{4}\sqrt{y_i^2}\right)A_4^{\rm tree}, \quad I_C\left[A_6^{\rm tree}\right] = \left(\prod_{i=1}^{4}\sqrt{y_i^2}\right)A_6^{\rm tree}$$

OSp(6|4) Yangian symmetry Bargeer, Biesert, Loebert, Meneghelli
The dual space is

$$P \to x, \quad Q^{I\alpha} \to \theta^{I\alpha}, \quad R^{IJ} \to y^{IJ}$$

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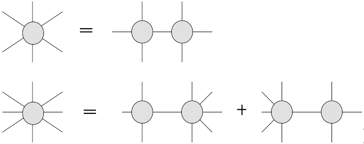
Establishing a recursion formula leads to all tree proof: Gang, Koh, Lee, Lipstein, me



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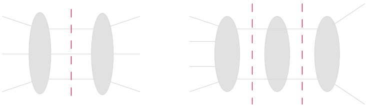

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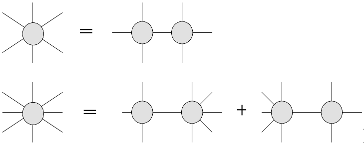

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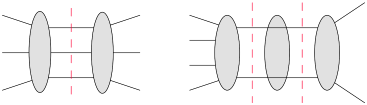

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At weak coupling ABJM is as dual superconformal as N=4 SYM
What about the regulated theory?
Is there also a Wilson-loop amplitude duality?

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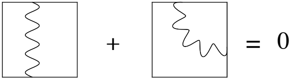
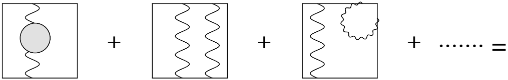
ABJM Amplitude/Wilson-Loops

The four-cusp Wilson loop: Henn, Plefka, Weigandt





$$\langle W_4^{ABJM} \rangle = 1 + \frac{N^2}{k^2}\left[-\left(\frac{(-\mu^2 x_{13}^2)^{2\epsilon}}{(2\epsilon)^2} + \frac{(-\mu^2 x_{24}^2)^{2\epsilon}}{(2\epsilon)^2} \right) + \frac{1}{2}\ln^2\left(\frac{x_{13}^2}{x_{24}^2} \right) + \text{const.} \right]$$

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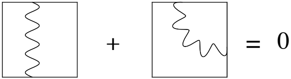

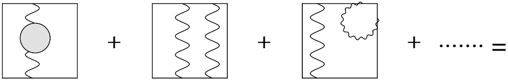

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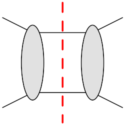
One-loop puzzle: Chen, Me



$\neq \; 0$

Dual conformal invariance tells us

$$
\begin{aligned}
I_4^{1-\text{loop}} &= \int \mathcal{D}^3 X_5 \frac{\epsilon(5,1,2,3,4)}{X_{51}^2 X_{52}^2 X_{53}^2 X_{54}^2} \\
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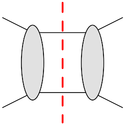

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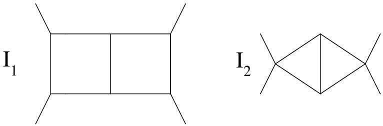

$$I_1^{2-\text{Loop}} \equiv \int \mathcal{D}^3X_5\mathcal{D}^3X_6 \frac{16\epsilon(5,1,2,3,4)\epsilon(6,1,2,3,4)}{X_{51}^2X_{53}^2X_{54}^2X_{56}^2X_{61}^2X_{63}^2X_{62}^2X_{42}^2}$$

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$$\mathcal{A}_4^{2-\text{loop}} = -\frac{1}{16\pi^2}\left(\frac{N}{K}\right)^2 \mathcal{A}_4^{\text{tree}}\left[\frac{(-s/\tilde{\mu}^2)^{-2\epsilon}}{(2\epsilon)^2} + \frac{(-t/\tilde{\mu}^2)^{-2\epsilon}}{(2\epsilon)^2} - \frac{1}{2}\log^2\left(\frac{-s}{-t}\right) + a + \mathcal{O}(\epsilon)\right]$$

Feynman diagrams: Bianchi, Leoni, Mauri, Penati, Santambrogio

ABJM Amplitude/Wilson-Loops

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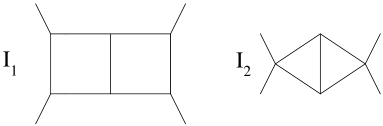

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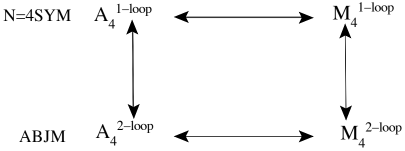

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ABJM Amplitude/Wilson-Loops

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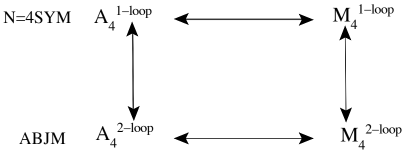

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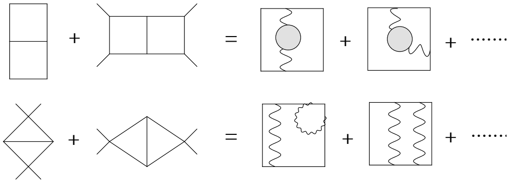

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Extensions:

- $\mathrm{N = 4SYM\ W_n^{1-loop} = ABJM\ \ W_n^{2-loop}}$ has been numerically checked to *n*-point: Weigandt
- All loop duality: BDS a solution to "both" N=4 SYM and ABJM?

$$\mathrm{Log}\frac{\mathrm{A_4}}{\mathrm{A_4^{tree}}} = [\mathrm{IR\ div}] + \frac{\mathrm{f_{SYM}}(\lambda)}{4}\mathrm{Log}^2\left(\frac{\mathrm{s}}{\mathrm{t}}\right) + \mathrm{const.} + \mathcal{O}(\epsilon)$$

The answer is correct for ABJM if

$$f_{CS}(\lambda) = \frac{1}{2}f_{SYM}(\lambda)\Big|_{\frac{\sqrt{\lambda}}{4\pi}\to h(\lambda)}$$

The correspondence matches to all order in ϵ Bianchi, Leoni, Penati

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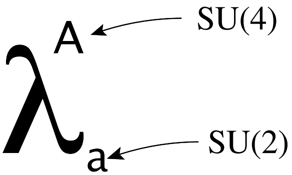
Six-dimensions

D=6



Three-point kinematics

4D: $s_{ij} = \langle ij\rangle[ji] = 0 \rightarrow \begin{array}{l} [ij]=0, \ \ \langle ij\rangle \neq 0 \\ \langle ij\rangle = 0, \ \ [ij]\neq 0 \end{array}$

6D: $\langle i_a|j_{\dot{a}}] \equiv \lambda_i^A{}_a \tilde{\lambda}_{jA\dot{a}}$

$$\langle i_a|j_{\dot{a}}] = \left(\begin{array}{cc} [ij] & 0 \\ 0 & -\langle ij\rangle \end{array} \right)$$

$$det\left(\langle i_a|j_{\dot{a}}]\right) = 0 \rightarrow \quad \langle i_a|j_{\dot{a}}] = u_i \tilde{u}_j$$

$$u_i^a w_{ia} = 1, \quad \tilde{u}_i^a \tilde{w}_{ia} = 1$$

- Three point amplitude: $A(u_i, \tilde{u}_i, w_i, \tilde{w}_i)$
- Invariant under: [C. Cheung, D O'Connell (08)]

$$u_i \rightarrow \alpha u_i, \ \tilde{u}_i \rightarrow \alpha^{-1}\tilde{u}_i, \ w_i \rightarrow \alpha^{-1} w_i, \ \tilde{w}_i \rightarrow \alpha\tilde{w}_i$$

- Invariant under

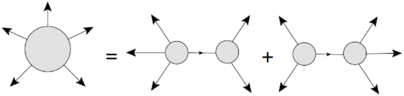
$$w_i \rightarrow w_i + b_i u_i, \ \tilde{w}_i \rightarrow \tilde{w}_i + c_i \tilde{u}_i$$

$\mathcal{N} = (1,1) SYM$

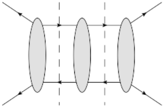
$$A_3 = (u_{1a}u_{2b}w_{3c} + u_{1a}w_{2b}u_{3c} + w_{1a}u_{2b}u_{3c})(\tilde{u}_{1a}\tilde{u}_{2b}\tilde{w}_{3c} + \tilde{u}_{1a}\tilde{w}_{2b}\tilde{u}_{3c} + \tilde{w}_{1a}\tilde{u}_{2b}\tilde{u}_{3c})$$

$$A_4 = \delta^6(P)\frac{\delta^4(Q)\delta^4(\tilde{Q})}{st} \equiv \delta^6(P)\delta^4(Q)\delta^4(\tilde{Q})f_4$$

$$f_4 = \frac{1}{x_{13}^2 x_{24}^2} \to I[f_4] = x_1^2 x_2^2 x_3^2 x_4^2 f_4$$



$$\to \quad I[f_n] = \prod_i^n x_i^2 f_n$$



$$\to I[I_n^L] = (x_{\ell_1}^2 \cdots x_{\ell_L}^2)^4 \prod_i^n x_i^2 I_n^L$$

Dennen, me

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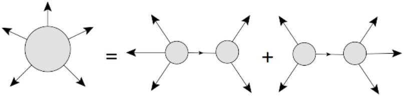

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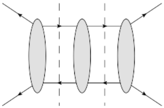

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Dennen, me

$\mathcal{N} = (1,1)SYM$

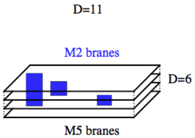
- 6D N=(1,1) SYM is Dual conformal symmetry without conformal symmetry Dennen, me
- True for tree level maximal SYM in arbitrary D. Caron-Hout, O'Connell

$\mathcal{N} = (2,0)$

On-shell SU(2): $\left(\psi^{Ia}[8],\ B_{(ab)}[3],\ \phi^{[IJ]}[5]\right)$

$$\begin{aligned}\Phi(\eta^a,\hat\eta^a) &= \phi + \eta^a\psi_a + \hat\eta^a\psi'_a + \eta^2\phi' + \hat\eta^2\phi'' + (\eta^a\hat\eta_a)\phi''' + \eta_{(a}\hat\eta_{b)}B^{(ab)} \\ &+ \hat\eta^2\eta^b\bar\psi_b + \eta^2\hat\eta^b\bar\psi'_b + \eta^2\hat\eta^2\phi''''\end{aligned}$$

On-shell SUSY: $\eta^b = (\frac{1}{2},0)$ and $\hat\eta^b = (0,\frac{1}{2})$ of Sp(2)×Sp(2)∈Sp(4)



Conformal amplitudes does not exists (Huang, Lipstein [10])

What is the minimum symmetry one can impose to obtain an on-shell amplitude?

$\mathcal{N} = (2,0)$

Is there a $\langle B_{ab}(1)B_{cd}(2)B_{cd}(3)\rangle$

- Litte group: A_3 polynomial of degree 6 in (u, w)
- α-scale invariance: $A_3 \sim uuuwww$
- β-shift invariance: No solutions

Little group+Lorentz invariance$\rightarrow$ forbids three-point interaction for self-dual tensors

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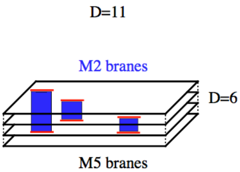
$\mathcal{N} = (2,0)$

Tensors have to couple to particles of other spin:

$$\langle B_{(ad)}(1)A_{bd\dot{2}}(2)A_{(c\dot{f})}(3)\rangle = (u_{1a}u_{2b}w_{3c} + u_{2b}u_{3c}w_{1a} + u_{3c}u_{1a}w_{2b})u_{1d}\tilde{u}_{2\dot{e}}\tilde{u}_{3\dot{f}}$$

$\rightarrow$ The degrees of freedom are incorrect?



Look for $\mathcal{N} = (2,0), (1,0)$ SUSY involving self-dual tensors

$\mathcal{N} = (2,0)$

We now look for general $\mathcal{A}_3(u, w, \tilde{u}, \tilde{w}, \eta, \hat{\eta})$
On-shell 8+8 SUSY charges:

$$Q^{A-} = \lambda^{Aa}\frac{\partial}{\partial\eta^a}, \quad Q^{A+} = \lambda^{Aa}\eta_a, \quad \hat{Q} = Q/.\{\eta :> \hat{\eta}\}$$

- Sp(2)×Sp(2) R-symmetry invariance: $\mathcal{A}_n \sim \eta^n\hat{\eta}^n$
- SUSY: $Q^{A-}, Q^{A-}\mathcal{A}_n = 0 \rightarrow \mathcal{A}_n \sim \Delta(Q)f(\eta, \hat{\eta})$

$$\Delta(Q) \equiv (\mathbf{u}_1\mathbf{u}_2 + \mathbf{u}_2\mathbf{u}_3 + \mathbf{u}_3\mathbf{u}_1)(\sum_{i=1}^{3}\mathbf{w}_i), \quad \mathbf{u}_i = u_i{}^a\eta_{ia}$$

- α-scaling invariance: $f(\eta, \hat{\eta}) \rightarrow \alpha^{-1}f(\eta, \hat{\eta})$

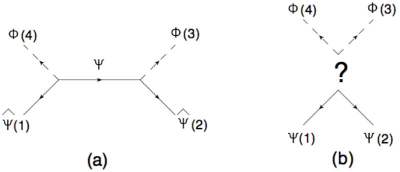
$\mathcal{N} = (2,0)$

Look for $\mathcal{N} = (2,0), (1,0)$ SUSY involving self-dual tensors:

- No $\mathcal{A}_3$ involving two tensors $B_{\mu\nu}$ (unless graviton)
- Non-gravitation $\mathcal{A}_3$ involving one tensor:

$$\mathcal{L}_3 \sim \epsilon^{\mu\nu\rho\sigma\tau\upsilon} B_{\mu\nu} F_{\rho\sigma} F_{\tau\upsilon}$$

Appears in supergravity actions Can this be self-consistent without gravitation degrees of freedom?

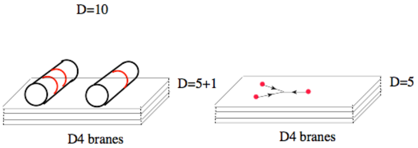


$\mathcal{N} = (2, 0)$

- Imposing vanishing factorization on the four-point, $\mathcal{A}_4 = \delta^6(P)\delta^4(Q)\delta^4(\hat{Q})$

$$A_4 \sim H^4$$

- One can reduce to D=5, $SU(4) \to USp(2, 2)$ allows $\langle i_a | j_b \rangle$:



$\rightarrow$No three-point KK coupling

- The only non-vanishing three-point is two (2,0) KK modes with one massless vector

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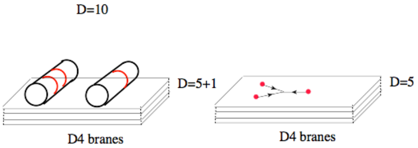

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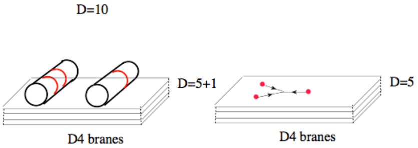

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Conclusion

- Un expect Dual Super Conformal invariance of ABJM
- All loop equivalence between four-point 4D N=4 SYM / 3D ABJM
- Dual conformal without conformal in 6D
- Sharpening the difficulty of interacting self-dual tensors theories in 6D.