A Higher Derivative Extension of the Salam-Sezgin Model from Superconformal Methods

Frederik Coomans

KU Leuven

Workshop on Conformal Field Theories Beyond Two Dimensions

16/03/2012, Texas A&M

Based on work with E. Bergshoeff, E. Sezgin and A. Van Proeyen

Introduction

Possible SUGRA theories for a given spacetime dimension and number of supersymmetry generators [Van Proeyen, Freedman, 2012]

D		32		24	20	16		12	8	4
11		M								
10		IIA	IIB			I				
9		N=2				N=1				
8		N=2				N=1				
7		N=4				N=2				
6		(2,2)		(2,1)		(1,1)	(2,0)		(1,0)	
5		N=8		N=6		N=4			N=2	
4		N=8		N=6	N=5	N=4		N=3	N=2	N=1

Introduction

Method

For $D = 4, 5, 6$ and $\#Q \leq 16$ (i.e when there are matter multiplets) matter-coupled SUGRA actions can be constructed via **superconformal tensor calculus** (SCTC)

What? Use superconformal symmetry as a **tool** to construct SUGRA theories (which are only invariant under super Poincaré). Construct superconformal theory by coupling a **compensating matter multiplet** to the **Weyl multiplet** (gauge multiplet of the superconformal group). Compensator multiplet 'compensates' for the redundant conformal symmetries.

Introduction

Method

For $D = 4, 5, 6$ and $\#Q \leq 16$ (i.e when there are matter multiplets) matter-coupled SUGRA actions can be constructed via **superconformal tensor calculus** (SCTC)

Why? (advantages over other methods like Noether method, superspace, ...)

- Conformal symmetry severely restricts # couplings that can be written
- Extra symmetry gives inside in the structure of the theory
- Different compensators give rise to different formulations of the Poincaré theory
- Easy to construct off-shell actions

Introduction

Use of SCTC for minimal $D = 6$ SUGRA:

- [Bergshoeff, Sezgin, Van Proeyen, 1986]: Weyl & matter multiplets, action formulas
- [Van Proeyen, FC, 2011]: Complete off-shell pure SUGRA action
- [Bergshoeff, Rakowski, 1987 & Bergshoeff, Sezgin, Salam, 1987]: Off-shell supersymmetric Riem^2 action

In this talk: [Bergshoeff, Sezgin, Van Proeyen, FC, 2012]:

- **Gauge** $\mathrm{U}(1)$ **R-symmetry** of pure theory
- Add Riem^2 action and study gauging procedure **in presence of higher derivative terms**
- Study **solutions of gauged higher derivative action**

Introduction

Why interest in higher-derivative terms?

- Appear as α' corrections in effective action of string theory
- Corrections to black hole entropy (much progress in $D = 4, 5$ not yet in $D = 6$)
- Compactification to $D = 3$: make graviton a (massive) propagating mode (e.g. [Bergshoeff et al, 2010])

Outline

1. **SUSY in $D = 6$**
2. **Construction of actions via superconformal tensor calculus**

   -Construction of the pure action
   -Coupling to a vector multiplet and gauging of the R-symmetry
   -Connection to Salam-Sezgin
   -Construction of R^2-action
   -The total Lagrangian

3. **Vacuum solutions of the gauged $R + R^2$ Lagrangian**
4. **Conclusions and outlook**

1. SUSY in $D = 6$

Spinors in minimal $D = 6$ SUGRA

- Irreducible spinor has 8 real components
- Is Weyl spinor: $\lambda = P_L \lambda$ or $\lambda = P_R \lambda$
- Symplectic Majorana condition: $\lambda^i = \epsilon^{ij} (\lambda^j)^C$
- Minimal SUSY algebra has 8 supercharges (like $\mathcal{N} = 2$ in $D = 4$)
- Pair of SUSY generators $Q^i_\alpha = \{Q^1_\alpha, Q^2_\alpha\}$ of the same chirality, hence $\mathcal{N} = (1, 0)$
- R-symmetry group is $\mathrm{SU}(2)$
- Transformations between the SUSY parameters $\epsilon^i = \{\epsilon^1, \epsilon^2\}$ preserving the symplectic structure are $\mathrm{SU}(2)$ transformations

p-form gauge fields in minimal $D = 6$ SUGRA

$$S_p = -\frac{1}{2}\int {}^*F^{(p+1)} \wedge F^{(p+1)}\,, \qquad F^{(p+1)} = dA^{(p)}$$

- Reducible gauge symmetry $\delta A^{(p)} = d\theta^{(p-1)}$

  $\longrightarrow$ Watch out when counting degrees of freedom!

- Degrees of freedom
  - **Off-shell**: as antisymmetric tensor in SO(5)
  - **On-shell**: as antisymmetric tensor in SO(4)

- p-forms are dual to $(D - p - 2) = (4 - p)$-forms, hence 2-forms are selfdual

Off-shell vs on-shell multiplets

SUSY theories are built up from SUSY **multiplets** (i.e. field representations of the SUSY algebra)

- **off-shell multiplets**: SUSY-algebra closes on the fields of the multiplet; # off-shell bosonic d.o.f. = # off-shell fermionic d.o.f.
- **on-shell multiplets**: SUSY-algebra only closes on the fields of the multiplet modulo EOM; # on-shell bosonic d.o.f. = # on-shell fermionic d.o.f.

Sum of two off-shell actions is again off-shell supersymmetric; no modification of the SUSY rules necessary!

2. Construction of actions via superconformal tensor calculus

Gravity as a conformal gauge theory

P_a	M_{ab}	D	K_a
ξ^a	λ^{ab}	λ_D	λ_K^a
$e_\mu{}^a$	$\omega_\mu{}^{ab}$	b_μ	$f_\mu{}^a$

Constraints determine two gauge fields

$$R_{\mu\nu}(P^a) = 0 \implies \omega_\mu{}^{ab} = \omega_\mu{}^{ab}(e, b)$$
$$e^\nu{}_b R_{\mu\nu}(M^{ab}) = 0 \implies f_\mu{}^a = f_\mu{}^a(e, b)$$

'Weyl multiplet': $e_\mu{}^a$, b_μ

Gravity as a conformal gauge theory

Use scalar field ϕ as **compensator**

Conformal gravity:

$$\mathcal{L}_C = -\sqrt{g}\phi\Box^C\phi = -\sqrt{g}\phi\Box\phi - \frac{1}{6}\sqrt{g}R\phi^2 + \dots$$

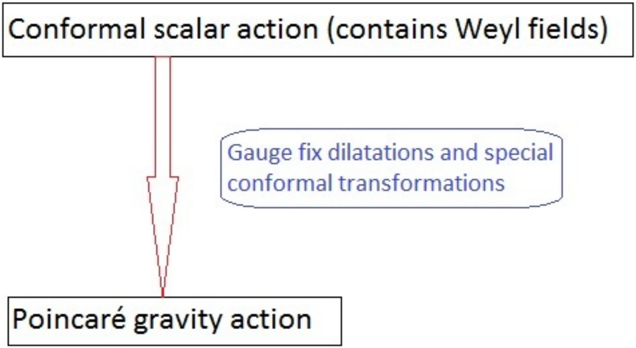
Gauge fixing,

- $\delta(\lambda_K)b_\mu = e_{\mu a}\lambda_K^a \Longrightarrow$ special conformal gauge fixing: $b_\mu = 0$,
- $\delta(\lambda_D)\phi = \lambda_D\phi \Longrightarrow$ dilatational gauge fixing: $\phi = \sqrt{3}M_P$,

leads to EH action:

$$\mathcal{L} = -\frac{M_P^2}{2}\sqrt{g}R$$

Gravity as a conformal gauge theory



Minimal $D = 6$ SUGRA

P_a	M_{ab}	D	K_a	$\mathrm{SU}(2)$	Q^i	S^i
ξ^a	λ^{ab}	λ_D	λ_K^a	Λ^{ij}	ϵ^i	η^i
$e_\mu{}^a$	$\omega_\mu{}^{ab}$	b_μ	$f_\mu{}^a$	$\mathcal{V}_\mu^{ij}$	$\psi_\mu{}^i$	$\phi_\mu{}^i$

Constraints determine $\omega_\mu{}^{ab}$, $f_\mu{}^a$ and $\phi_\mu{}^i$ in terms of the others

Weyl multiplet: $e_\mu{}^a, b_\mu, \mathcal{V}_\mu^{ij}, \sigma, B_{\mu\nu}, \psi_\mu{}^i, \psi^i$

PS: there is also another choice of extra fields, i.e. another Weyl multiplet, but this one is chosen to obtain an invariant action

Minimal $D = 6$ SUGRA

Compensating multiplet: **linear multiplet** (off-shell, SC action known)

Field	Off-shell dof	On-shell dof
L^{ij}	3	3
$E_{\mu\nu\rho\sigma}$	5	1
φ^i	8	4

We know: superconformal action for coupling of vector and linear multiplet **plus** embedding of linear into vector multiplet $\Longrightarrow$ We know: superconformal action for linear multiplet

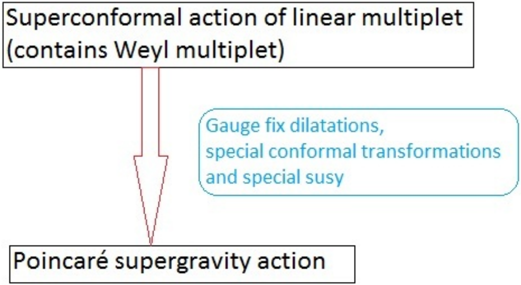
Gauge fixing: $\boxed{L_{ij} = \frac{1}{\sqrt{2}}\delta_{ij}\ , \quad \varphi^i = 0\ , \quad b_\mu = 0}$

$\longrightarrow$ fixes $D, SU(2)/U(1), K, S$

Minimal $D = 6$ SUGRA

Weyl × Linear		Gauge fixing		off-shell Poincaré		
${e_\mu}^a$ (15)	P_a, M_{ab}			${e_\mu}^a$ (15)	P_a, M_{ab}	9
b_μ (0)	K_a	$b_\mu = 0$	K_a	$*$		
${\mathcal{V}_\mu}^i{}_j$ (15)	SU(2)			${\mathcal{V}_\mu}^i{}_j$ (17)	SO(2)	
σ (1)				σ (1)		1
$B_{\mu\nu}$ (10)	Λ_μ			$B_{\mu\nu}$ (10)	Λ_μ	6
dilatations (−1)				$*$		
L^{ij} (3)		$L^{ij} = \frac{1}{\sqrt{2}}\delta^{ij}$	D, SU(2)/SO(2)	$*$		
$E_{\mu\nu\rho\sigma}$ (5)	$\tilde{\Lambda}_{\mu\nu\rho}$			$E_{\mu\nu\rho\sigma}$ (5)	$\tilde{\Lambda}_{\mu\nu\rho}$	
48				**48**		**16**
${\psi_\mu}^i$ (40)	Q^i			${\psi_\mu}^i$ (40)	Q^i	12
ψ^i (8)				ψ^i (8)		4
S-susy (−8)				$*$		
φ^i (8)		$\varphi^i = 0$	S^i	$*$		
48				**48**		**16**

Minimal $D=6$ SUGRA



$$
\begin{aligned}
e^{-1}\mathcal{L}_R\big|_{L=1} \quad = \quad & \frac{1}{2}R - \frac{1}{2}\sigma^{-2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{24}\sigma^{-2}F_{\mu\nu\rho}(B)F^{\mu\nu\rho}(B) + \mathcal{V}'_{\mu ij}\mathcal{V}'^{\mu ij} - \frac{1}{4}E^\mu E_\mu \\
& + \frac{1}{\sqrt{2}}E^\mu\mathcal{V}_\mu - \frac{1}{2}\bar{\psi}_\mu\gamma^{\mu\nu\rho}D_\nu\psi_\rho - 2\sigma^{-2}\bar{\psi}\gamma^\mu D'_\mu\psi + \ldots
\end{aligned}
$$

[Van Proeyen, FC, 2011] PS: we split the gauge field $\mathcal{V}_\mu^{ij} = \mathcal{V}'^{ij}_\mu + \frac{1}{2}\delta^{ij}\mathcal{V}_\mu$ into traceless plus trace and denote E_μ as the dual of the 4-form field strength

Gauging the theory

To obtain R-symmetry gauging we add a **vector multiplet**:

Field	Off-shell dof	On-shell dof
W_μ	5	4
Y^{ij}	3	0
Ω^i	8	4

-superconformal invariant action (includes also fields of the Weyl multiplet)

$$e^{-1}\mathcal{L}_{\rm V} = \sigma\Big(-\frac{1}{4}F_{\mu\nu}(W)F^{\mu\nu}(W) - 2\bar{\Omega}\gamma^\mu D'_\mu(\omega)\Omega + Y^{ij}Y_{ij}\Big) - \frac{1}{16}e^{-1}\varepsilon^{\mu\nu\rho\sigma\lambda\tau}B_{\mu\nu}F_{\rho\sigma}(W)F_{\lambda\tau}(W) + \dots$$

-coupling with linear multiplet

$$e^{-1}\mathcal{L}_{\rm VL} = Y_{ij}L^{ij} + 2\bar{\Omega}\varphi - L^{ij}\bar{\psi}_{\mu i}\gamma^\mu\Omega_j + \frac{1}{2}W_\mu\left(E^\mu - \bar{\psi}_\nu\gamma^{\nu\mu}\varphi\right)$$

[Bergshoeff, Sezgin, Van Proeyen, 1986]

Gauging the theory

$$\mathcal{L}_1 = (\mathcal{L}_R + \mathcal{L}_V + g\mathcal{L}_{VL})|_{L=1}$$

- Contains terms $\sigma Y_{ij} Y^{ij} + \frac{g}{\sqrt{2}} \delta^{ij} Y_{ij} \longrightarrow V = \frac{1}{4} g^2 \sigma^{-1}$ (after solving for Y^{ij})
- Has $\mathrm{U}(1)_R \times \mathrm{U}(1)$ gauge symmetry
- $E_{\mu\nu\rho\sigma}$ field equation $\Rightarrow E_\mu = \partial_\mu \phi + \sqrt{2}\mathcal{V}_\mu + gW_\mu$
- $\mathcal{V}_\mu$ field equation $\Rightarrow E_\mu +$ (fermion bilinears) $= 0$
- $\delta_{gauge}\phi = -\sqrt{2}\lambda - g\eta$, hence fixing the scalar $\phi = \phi_0$ implies
  - $\mathrm{U}(1)_R \times \mathrm{U}(1) \to \mathrm{U}(1)_R^{\mathrm{diag}}$
  - $\sqrt{2}\mathcal{V}_\mu + gW_\mu = 0$ (bosonically)

To the Salam-Sezgin model ...

'U(1) gauged Einstein-Maxwell SUGRA in $D = 6$, spontaneously compactifies on $\text{Mink}_4 \times S^2$ breaking half of the supersymmetries' [Salam, Sezgin, 1984]

$$e^{-1}\mathcal{L}_{SS} = \frac{1}{2}R - \frac{1}{2}\sigma^{-2}\partial_a\sigma\partial^a\sigma - \frac{1}{4}g^2\sigma^{-1} - \frac{1}{24}\sigma^2 G_{\mu\nu\rho}G^{\mu\nu\rho} - \frac{1}{4}\sigma F_{\mu\nu}(W)F^{\mu\nu}(W) + \ldots$$

- on-shell model
- U(1) vector appears inside the 2-form field strength:

$$G^{(3)} = d\widetilde{B}^{(2)} + dW^{(1)} \wedge W^{(1)}$$

How does our model, described by $\mathcal{L}_1$, relate to the SS model?

To the Salam-Sezgin model ...

After elimination of auxiliary fields our model becomes

$$
\begin{aligned}
e^{-1}\mathcal{L}_{\text{on-shell}} \quad = \quad & \frac{1}{2}R - \frac{1}{2}\sigma^{-2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{4}g^2\sigma^{-1} - \frac{1}{24}\sigma^{-2}F_{\mu\nu\rho}(B)F^{\mu\nu\rho}(B) \\
& -\frac{1}{4}\sigma F_{\mu\nu}(W)F^{\mu\nu}(W) + \frac{1}{24}e^{-1}\varepsilon^{\mu\nu\rho\sigma\lambda\tau}F_{\mu\nu\rho}(B)F_{\lambda\tau}(W)W_\sigma + \dots
\end{aligned}
$$

To obtain the SS-model we have to dualize the 2-form $B_{\mu\nu}$. We can do this by

- adding a Lagrange multiplier $\frac{1}{24}\varepsilon^{\mu\nu\rho\sigma\lambda\tau}F_{\mu\nu\rho}(B)\partial_\sigma\widetilde{B}_{\lambda\tau}$
- solving for $F_{\mu\nu\rho}(B)$

An alternative off-shell formulation

Other gauge fixing,

$$\boxed{\sigma = 1 \,, \quad L_{ij} = \frac{1}{\sqrt{2}} \delta_{ij} L \,, \quad \psi^i = 0 \,, \quad b_\mu = 0}$$

amounts to

$$\mathcal{L}_2 = (\mathcal{L}_R + \mathcal{L}_V + g\mathcal{L}_{VL})|_{\sigma=1}$$

Essentially: σ and ψ^i replaced by L and φ^i

The Riem^2 invariant

- Trick from [Bergshoeff, Rakowski, 1987 & Bergshoeff, Sezgin, Salam, 1987]: **Embed Weyl multiplet in non-Abelian vector multiplet**

- Denote $\omega_{-\mu}{}^{ab} = \omega_\mu{}^{ab} - \frac{1}{2} F_\mu{}^{ab}(B)$ (bosonic torsion)

- In the gauge '$\sigma = 1$':

$$\boxed{\left(-2\widehat{\omega}_{-\mu}{}^{ab}, -\widehat{R}^{abi}(Q), -2\widehat{F}^{abij}(\mathcal{V})\right) \sim \left(W_\mu{}^I, \Omega^{Ii}, Y^{Iij}\right)}$$

- Lagrangian for the non-Abelian vector multiplet is known [Bergshoeff, Sezgin, Van Proeyen, 1986]:

$$\begin{aligned} e^{-1}\mathcal{L}_{\rm YM}\big|_{\sigma=1} \quad = \quad & -\frac{1}{4} F_{\mu\nu}{}^I(W) F^{\mu\nu I}(W) - 2\bar{\Omega}^I \gamma^\mu D'_\mu(\omega)\Omega^I + Y^{Iij} Y^I_{ij} \\ & -\frac{1}{16} e^{-1} \varepsilon^{\mu\nu\rho\sigma\lambda\tau} B_{\mu\nu} F^I_{\rho\sigma}(W) F^I_{\lambda\tau}(W) \\ & +\frac{1}{2} F_{\nu\rho}{}^I \bar{\Omega}^I \gamma^\mu \gamma^{\nu\rho} \psi_\mu + \frac{1}{12} F_{\mu\nu\rho}(B) \bar{\Omega}^I \gamma^{\mu\nu\rho} \Omega^I \end{aligned}$$

The Riem^2 invariant

- Making the substitution amounts to

$$
\begin{aligned}
e^{-1}\mathcal{L}_{\mathrm{R}^2}\big|_{\sigma=1} \quad = \quad & R_{\mu\nu}{}^{ab}(\omega_-)R^{\mu\nu}{}_{ab}(\omega_-) - 2F^{ab}(\mathcal{V})F_{ab}(\mathcal{V}) - 4F'^{\,abij}(\mathcal{V})F'_{abij}(\mathcal{V}) \\
& + \frac{1}{4}e^{-1}\varepsilon^{\mu\nu\rho\sigma\lambda\tau}B_{\mu\nu}R_{\rho\sigma}{}^{ab}(\omega_-)R_{\lambda\tau\,ab}(\omega_-) \\
& + 2\bar{R}_{+ab}(Q)\gamma^\mu D_\mu(\omega,\omega_-)R_+^{ab}(Q) - R_{\nu\rho}{}^{ab}(\omega_-)\bar{R}_{+ab}(Q)\gamma^\mu\gamma^{\nu\rho}\psi_\mu \\
& - 8F'_{\mu\nu}{}^{ij}(\mathcal{V})\left(\bar{\psi}_i^\mu\gamma_\lambda R_{+j}^{\lambda\nu}(Q) + \frac{1}{6}\bar{\psi}_i^\mu\gamma\cdot F(B)\psi_j^\nu\right) \\
& - \frac{1}{12}\bar{R}_+^{ab}(Q)\gamma\cdot F(B)R_{+ab}(Q) - \frac{1}{2}\Big[D_\mu(\omega_-,\Gamma_+)R^{\mu\rho ab}(\omega_-) \\
& - 2F_{\mu\nu}{}^{\rho}(B)R^{\mu\nu ab}(\omega_-)\Big]\bar{\psi}_a\gamma_\rho\psi_b
\end{aligned}
$$

The total Lagrangian

$$\mathcal{L}_{\rm tot} = (\mathcal{L}_R + \mathcal{L}_V + g\mathcal{L}_{VL} - \frac{1}{8M^2}\mathcal{L}_{R^2})|_{\sigma=1}$$

Off-shell: every term is seperately invariant! (no $1/M^2$ corrections to the transformation rules)

$$\begin{aligned} e^{-1}\mathcal{L}_{\rm tot} \;=\;& \frac{1}{2}LR + \frac{1}{\sqrt{2}}gL\delta^{ij}Y_{ij} + Y^{ij}Y_{ij} + \frac{1}{2}L^{-1}\partial_\mu L\partial^\mu L - \frac{1}{24}LF_{\mu\nu\rho}(B)F^{\mu\nu\rho}(B) \\ & + L\mathcal{V}'^{k\ell}_a\mathcal{V}'^a{}_{k\ell} - \frac{1}{4}L^{-1}E_\mu E^\mu + \frac{1}{\sqrt{2}}E^\mu(\mathcal{V}_\mu + \frac{1}{\sqrt{2}}gW_\mu) \\ & - \frac{1}{4}F_{\mu\nu}(W)F^{\mu\nu}(W) - \frac{1}{16}e^{-1}\varepsilon^{\mu\nu\rho\sigma\lambda\tau}B_{\mu\nu}F_{\rho\sigma}(W)F_{\lambda\tau}(W) \\ & - \frac{1}{8M^2}\Big[R_{\mu\nu}{}^{ab}(\omega_-)R^{\mu\nu}{}_{ab}(\omega_-) - 2F^{ab}(\mathcal{V})F_{ab}(\mathcal{V}) - 4F^{abij}(\mathcal{V}')F_{abij}(\mathcal{V}') \\ & + \frac{1}{4}e^{-1}\varepsilon^{\mu\nu\rho\sigma\lambda\tau}B_{\mu\nu}R_{\rho\sigma}{}^{ab}(\omega_-)R_{\lambda\tau\,ab}(\omega_-)\Big] + \ldots \end{aligned}$$

3. Vacuum solutions

Perturbative or Toy Model

$\mathcal{L}_{R^2}$ contains kinetic terms for some auxiliaries! Can we still eliminate them?

1. Consider $1/M^2$ as a small parameter and eliminate auxiliaries perturbatively
   - **SUSY only order by order** in parameter $1/M^2$
   - **Open question**: does this on-shell Lagrangian correspond to the one obtained by compactifying the effective heterotic string Lagrangian?
2. Consider $1/M^2$ as an arbitrary (not necessarily small) parameter
   - Propagating auxiliaries give rise to **ghosts**
   - Consider theory as a toy model with **exact SUSY**
   - We will take this approach when studying solutions of the theory

Vacuum solutions

Only consider **bosonic field equations** (background fermions vanish)

- Lagrangian without higher derivative terms
  - $\text{Mink}_4 \times S^2$, preserving half of the supersymmetries (Salam-Sezgin)
  - No Mink_6 or (A)dS solution
- Lagrangian with higher derivative terms
  - Elimination of $Y^{ij}, E_{\mu\nu\rho\sigma}$ still possible
  - Elimination of $\mathcal{V}'^{ij}_{\mu}, \mathcal{V}_{\mu}$ no longer possible since they acquired kinetic terms
  - Solutions without fluxes
  - Solutions with 2-form flux or 3-form flux

Vacuum solutions without fluxes

- Only non-vanishing fields are metric and $L = L_0$
- Mink_6 only a solution if we switch off gauging $g = 0$, since $R = g^2 L_0$
- For $g \neq 0$ only solutions of the form $M_D \times M_{6-D}$, i.e. consider Ansatz

$$R_{\mu\nu\rho\sigma} = n_1 g^2 L_0 \left(g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho}\right), \quad R_{pqrs} = n_2 g^2 L_0 (g_{pr} g_{qs} - g_{ps} g_{qr}),$$
$$L = L_0, \quad M^2 = n_3 g^2$$

Spacetime	n_1	n_2	n_3
$\text{Mink}_4 \times S^2$	0	1/2	1/2
$\text{dS}_4 \times T^2$	1/12	0	1/12
$\text{dS}_4 \times S^2$	1/14	1/14	1/14
$\text{Mink}_3 \times S^3$	0	1/6	1/6
$\text{dS}_3 \times T^3$	1/6	0	1/6
$\text{dS}_3 \times S^3$	1/12	1/12	1/12

- All these solutions are non-susy

Vacuum solutions with 2-form flux

- Consider solutions of the form $M_4 \times M_2$
- Metric and $L = L_0$ non-vanishing and $\mathcal{V}_\mu, W_\mu$ have fluxes on M_2
- Consider Ansatz

$$R_{\mu\nu} = 3a\, g_{\mu\nu} \ , \qquad R_{rs} = b\, g_{rs} \ , \qquad L = L_0 \ ,$$
$$F_{rs}(W) = c\,\sqrt{g_2}\,\varepsilon_{rs} \ , \qquad F_{rs}(\mathcal{V}) = -\frac{g}{\sqrt{2}}\, c\sqrt{g_2}\,\varepsilon_{rs} \ ,$$

  - $\text{Mink}_4 \times S^2$ **is still a solution**, preserving half of the supersymmetries!
  - Other solutions include $\text{AdS}_4 \times S^2$, $\text{dS}_4 \times S^2$, $\text{dS}_4 \times H^2$ (all non-susy)

Vacuum solutions with 3-form flux

- Consider solutions of the form $M_3 \times M_3$
- Metric and $L = L_0$ non-vanishing and $B_{\mu\nu}$ has fluxes on both M_3's
- Consider Ansatz

$$
\begin{aligned}
R_{\mu\nu}{}^{\rho\sigma} &= 2a\,\delta^{\rho}_{[\mu}\delta^{\sigma}_{\nu]}, \qquad R_{pq}{}^{rs} = 2b\,\delta^{r}_{[p}\delta^{s}_{q]}, \qquad L = L_0\ , \\
F_{\mu\nu\rho}(B) &= 2c\sqrt{-g_1}\varepsilon_{\mu\nu\rho}\ , \qquad F_{rst}(B) = -2c\sqrt{g_2}\varepsilon_{rst}
\end{aligned}
$$

  - $c^2 = -a = b$: $\mathrm{AdS}_3 \times S^3$, preserving full susy
  - Other solutions include $\mathrm{AdS}_3 \times S^3$, $\mathrm{dS}_3 \times S^3$, $\mathrm{dS}_3 \times H^3$ (all non-susy)

4. Conclusions and outlook

Conclusions

- Use of superconformal calculus to construct minimal $D = 6$ R-symmetry gauged supergravity with higher derivative ($\mathrm{Riem}^2 + \ldots$) terms
- All parts of the action seperately off-shell
- Auxiliaries can be eliminated perturbatively; correspondence with compactified string Lagrangian?
- Potential is not modified by Riem^2-terms (no couplings with Y^{ij} in $\mathcal{L}_{R^2}$)
- Supersymmetric $\mathrm{Mink}_4 \times S^2$ solution is still valid
- Also other solutions found

Outlook

- $D = 6$ is highest dimension that allows off-shell formulation: worthwile to investigate further
- Adding matter couplings (Yang-Mills multiplets, hypermultiplets)
- Anomalies (Lorentz CS term is part of the Riem^2-invariant)
- Computation of the spectrum in $\text{Mink}_4 \times S^2$
- Existence of other higher curvature invariants in $D = 6$?
- Higher derivative terms contribute corrections to the BH entropy; are important for connection microscopic/macroscopic entropy
  - In $D = 4, \mathcal{N} = 2$ [Lopes, Cardoso, de Wit, Mohaupt, 2004]
  - In $D = 5, \mathcal{N} = 2$ [de Wit, Katmadas, 2011]
- Need to find BH solutions !
  - so far only BH solutions for ungauged theory without higher derivative terms [Gibbons, Maeda, 1988]