A Gravity Realization of LCFT's in Two Dimensions and Beyond

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Why Higher-Curvature Gravity ?

- Obtain better quantum behaviour
- Test limits of the AdS/CFT correspondence

Problem: Gravity is perturbative non-renormalizable

$$\mathcal{L} \sim R + a \left(R_{\mu\nu}{}^{ab} \right)^2 + b \, (R_{\mu\nu})^2 + c \, R^2 \; :$$

renormalizable but not unitary

Stelle (1977)

massless spin 2 and massive spin 2 have opposite sign !

Special Cases

- In three dimensions there is no massless spin 2 !

$\Rightarrow$ "3D Topological Massive Gravity"

Deser, Jackiw, Templeton (1982)

. "3D New Massive Gravity"

Hohm, Townsend + E.B. (2009)

- Degeneracy for special point in parameter space

$\Rightarrow$ "3D Chiral Gravity"

Li, Song, Strominger (2008)

. "$D \geq 3$ Critical Gravity"

Lü and Pope (2011)

Logarithmic Conformal Field Theories (LCFT's)

- Einstein mode gets replaced by log mode

- At the same time CFT becomes a rank 2 LCFT

  Gurarie (1993), Flohr (2001), Gaberdiel (2001)

- This talk : Higher degeneracies give rise to higher-rank LCFT's

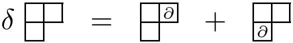
- Higher-Curvature Gravity theories can be constructed starting from FP equations and solving for differential subsidiary conditions

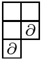
- This requires fields with zero massless degrees of freedom

Massless Degrees of Freedom

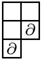
field $S \sim$ 

gauge parameters $\lambda_1 \sim$  $\lambda_2 \sim$ 

gauge transformation 

curvature $R(S) \sim$ 

Zero Massless D.O.F.

"Einstein tensor" $G(S) \sim \star\star$ 

Requirement : $G(S) \sim$  $\Rightarrow$ E.O.M. : $G(S) = 0$

$$s = 2 \;:\; p + q = D - 1$$

Example : $p = q = 1\,, D = 3\,,$ $S \sim$ 

"Boosting Up the Derivatives"

Second-order Derivative Generalized FP

Curtright (1980)

$$\left(\Box - m^2\right) S = 0\,, \qquad\qquad S^{\rm tr} = 0\,, \quad \partial \cdot S = 0$$

$$\partial \cdot S = 0 \quad \Rightarrow \quad S = G(T)$$

$$\left(\Box - m^2\right) G(T) = 0\,, \qquad\qquad G(T)^{\rm tr} = 0$$

Higher-derivative Gauge Theory

"Taking the Square Root"

Consider 1-forms in 3D , 3-forms in 7D , etc.

The KG-operator factorizes : $(\Box - m^2) = \mathcal{D}(m)\mathcal{D}(-m)$

Take the "square root" $\mathcal{D}(m)\, S = 0$

$$\mathcal{D}(m)\, G(T) = 0$$

Higher-derivative Gauge Theory

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3D Einstein-Hilbert Gravity

Deser, Jackiw, 't Hooft (1984)

There are no massless gravitons

Adding higher-derivative terms leads to "massive gravitons"

Fierz-Pauli

- $(\Box - m^2)\,\tilde{h}_{\mu\nu} = 0\,, \qquad \eta^{\mu\nu}\tilde{h}_{\mu\nu} = 0\,, \;\; \partial^\mu \tilde{h}_{\mu\nu} = 0$

- $\mathcal{L}_{\rm FP} = \frac{1}{2}\tilde{h}^{\mu\nu}G_{\mu\nu}(\tilde{h}) + \frac{1}{2}m^2\left(\tilde{h}^{\mu\nu}\tilde{h}_{\mu\nu} - \tilde{h}^2\right)\,, \qquad \tilde{h} \equiv \eta^{\mu\nu}\tilde{h}_{\mu\nu}$

no non-linear extension !

number of propagating modes is $\frac{1}{2}D(D+1) - 1 - D = \begin{cases} 5 & \text{for } 4D \\ 2 & \text{for } 3D \end{cases}$

Note : the numbers become 2 (4D) and 0 (3D) for $m = 0$

Higher-derivative Extension in 3D

$$\partial^\mu \tilde{h}_{\mu\nu} = 0 \quad \Rightarrow \quad \tilde{h}_{\mu\nu} = \epsilon_\mu{}^{\alpha\beta}\epsilon_\nu{}^{\gamma\delta}\partial_\alpha\partial_\gamma h_{\beta\delta} \equiv G_{\mu\nu}(h)$$

$$\left(\Box - m^2\right)\; G^{\text{lin}}_{\mu\nu}(h) = 0\,, \qquad R^{\text{lin}}(h) = 0$$

Non-linear generalization : $\quad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad \Rightarrow$

$$\mathcal{L} = \sqrt{-g}\left[-R - \frac{1}{2m^2}\left(R^{\mu\nu}R_{\mu\nu} - \frac{3}{8}R^2\right)\right]$$

"New Massive Gravity" : unitary !

Mode Analysis

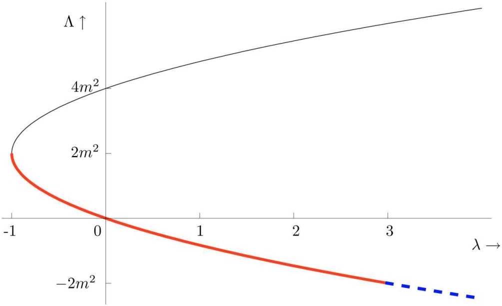
- Take NMG with metric $g_{\mu\nu}$, cosmological constant Λ and coefficient $\sigma = \pm 1$ in front of R

- lower number of derivatives from 4 to 2 by introducing an auxiliary field $f_{\mu\nu}$

- after linearization and diagonalization the two fields describe a massless spin 2 with coefficient $\bar{\sigma} = \sigma - \frac{\Lambda}{2m^2}$ and a massive spin 2 with mass $M^2 = -m^2\bar{\sigma}$

- special cases: 3D NMG and $D \geq 3$ "critical gravity" for special value of Λ

Unitary Bulk Region



$\sigma = -1$ $\lambda \neq \Lambda$!

$c = 0$ LCFT at cross-over point $\lambda = 3$

Topological Massive Gravity

Taking the Square Root

- $\left(\Box - m^2\right) \tilde{h}_{\mu\nu} = \left[\mathcal{O}(m)\mathcal{O}(-m)\right]_{\mu}{}^{\rho}\tilde{h}_{\rho\nu}$ with

$$\left[\mathcal{O}(\pm m)\right]_{\mu}{}^{\rho} = \epsilon_{\mu}{}^{\tau\rho}\partial_{\tau} \pm m\delta_{\mu}^{\rho}$$

Taking the Square Root

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$$\left[\mathcal{O}(\pm m)\right]_\mu{}^\rho = \epsilon_\mu{}^{\tau\rho}\partial_\tau \pm m\delta_\mu^\rho$$

- $m\tilde{h}_{\mu\nu} = \epsilon_\mu{}^{\rho\sigma}\partial_\rho \tilde{h}_{\sigma\nu}$: $\sqrt{\text{FP}}$

Taking the Square Root

- $(\Box - m^2)\,\tilde{h}_{\mu\nu} = \left[\mathcal{O}(m)\mathcal{O}(-m)\right]_{\mu}{}^{\rho}\tilde{h}_{\rho\nu}$ with

$$\left[\mathcal{O}(\pm m)\right]_{\mu}{}^{\rho} = \epsilon_{\mu}{}^{\tau\rho}\partial_{\tau} \pm m\delta_{\mu}^{\rho}$$

- $m\tilde{h}_{\mu\nu} = \epsilon_{\mu}{}^{\rho\sigma}\partial_{\rho}\tilde{h}_{\sigma\nu}$: $\sqrt{\mathrm{FP}}$

- $S = \frac{1}{2}\int d^3x\left(\epsilon^{\mu\nu\rho}\tilde{h}_{\mu}{}^{\sigma}\partial_{\nu}\tilde{h}_{\rho\sigma} + m(\tilde{h}^{\nu\mu}\tilde{h}_{\mu\nu} - \tilde{h}^2)\right)$

Aragone, Khoudeir (1986)

From $\sqrt{\mathrm{FP}}$ to Topological Massive Gravity

- $\sqrt{\mathrm{FP}}$: $\quad m\tilde{h}_{\mu\nu} = \epsilon_\mu{}^{\rho\sigma}\partial_\rho \tilde{h}_{\sigma\nu}$

From $\sqrt{\mathrm{FP}}$ to Topological Massive Gravity

- $\sqrt{\mathrm{FP}}$: $\quad m\tilde{h}_{\mu\nu} = \epsilon_\mu{}^{\rho\sigma}\partial_\rho\tilde{h}_{\sigma\nu}$

- $\partial^\mu\tilde{h}_{\mu\nu} = 0 \quad \Rightarrow \quad \tilde{h}_{\mu\nu} = G^{\mathrm{lin}}_{\mu\nu}(h)$

From $\sqrt{\mathrm{FP}}$ to Topological Massive Gravity

- $\sqrt{\mathrm{FP}}$: $m\tilde{h}_{\mu\nu} = \epsilon_\mu{}^{\rho\sigma}\partial_\rho \tilde{h}_{\sigma\nu}$

- $\partial^\mu \tilde{h}_{\mu\nu} = 0 \quad \Rightarrow \quad \tilde{h}_{\mu\nu} = G^{\mathrm{lin}}_{\mu\nu}(h)$

- Non-linear generalization : $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad \Rightarrow$

$$\mathcal{L} \sim -\sqrt{-g}\, R + \frac{1}{m}\mathcal{L}_{\mathrm{LCS}} \ : \qquad \mathrm{TMG}$$

Deser, Jackiw, Templeton (1982)

Generalization

Generalized Massive Gravity (GMG)

$$S[g] = \frac{1}{\kappa^2} \int d^3x \sqrt{-g} \left[\sigma R - 2\lambda m^2 + \frac{1}{m^2} K \right] + \frac{1}{\mu} \mathcal{L}_{\rm LCS}$$

$$m^2 = m_+ m_- \,, \qquad \mu = -\frac{m_+ m_-}{m_+ - m_-}$$

special cases:

Generalized Massive Gravity (GMG)

$$S[g] = \frac{1}{\kappa^2}\int d^3x\,\sqrt{-g}\left[\sigma R - 2\lambda m^2 + \frac{1}{m^2}K\right] + \frac{1}{\mu}\mathcal{L}_{\mathrm{LCS}}$$

$$m^2 = m_+ m_- \,, \qquad \mu = -\frac{m_+ m_-}{m_+ - m_-}$$

special cases:

- $m_+ = m_-$: NMG

Generalized Massive Gravity (GMG)

$$S[g] = \frac{1}{\kappa^2} \int d^3x \sqrt{-g} \left[\sigma R - 2\lambda m^2 + \frac{1}{m^2} K \right] + \frac{1}{\mu} \mathcal{L}_{\mathrm{LCS}}$$

$$m^2 = m_+ m_- \,, \qquad \mu = -\frac{m_+ m_-}{m_+ - m_-}$$

special cases:

- $m_+ = m_-$: NMG
- $m_+ \to \infty$: TMG

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3D critical gravity

At critical point : $\mathcal{D}(\pm m) \rightarrow \mathcal{D}_{L,R}$ or, shortly, L,R

- TMG : $O_{\text{kin}} \sim$ LLR $\leftrightarrow c_L = 0$ rank 2 LCFT for left-movers

  but $c_R \neq 0$ CFT for right-movers

- NMG : $O_{\text{kin}} \sim$ LLRR $\leftrightarrow c_L = c_R = 0$ rank 2 LCFT

- GMG : $O_{\text{kin}} \sim$ LLLR $\leftrightarrow c_L = 0$ rank 3 LCFT for left-movers

Critical Gravity beyond 3D

$$S = \frac{1}{\kappa^2} \int \mathrm{d}^D x \sqrt{-g} \left[\sigma R - 2\lambda m^2 + \frac{1}{m^2} G^{\mu\nu} S_{\mu\nu} + \frac{1}{m'^2} \mathcal{L}_{\mathrm{GB}} \right]$$

Lü and Pope (2011); Deser, Liu, Lü, Pope, Şişman and Tekin (2011)

$$R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} - 4 R^{\mu\nu} R_{\mu\nu} + R^2 = W^{\mu\nu\rho\sigma} W_{\mu\nu\rho\sigma} - 4(D-3) G^{\mu\nu} S_{\mu\nu}$$

after lowering the derivatives and linearization around AdS we find

$$\bar{\sigma}(\Lambda) \equiv \sigma - \frac{\Lambda}{m^2} \frac{1}{D-1} + 4 \frac{\Lambda}{m'^2} \frac{(D-3)(D-4)}{(D-1)(D-2)}$$

The critical point is defined by $\bar{\sigma}(\Lambda_{\mathrm{crit}}) = 0$

Massless and Log Modes

$D = 4$: massive -2 -1 0 $+1$ $+2$ $\Rightarrow$ massless -2 $+2$

$$\boxed{\mathcal{G}_{\mu\nu}\left(\mathcal{G}(h)\right) = 0} \qquad k_{\mu\nu} \equiv \mathcal{G}_{\mu\nu}(h) \quad \Rightarrow \quad \mathcal{G}_{\mu\nu}(k) = 0$$

$k_{\mu\nu} = 0$	trivial solution	massless modes
$k_{\mu\nu} = \nabla_{(\mu}A_{\nu)}$	"gauge solutions"	"Proca log modes"
$k^{\perp}_{\mu\nu} \neq \nabla_{(\mu}A_{\nu)}$	"non-gauge solutions"	"spin 2 log modes"

Unitarity

$$\langle \mathcal{O}^i \mathcal{O}^j \rangle \sim \begin{pmatrix} 0 & \mathrm{CFT} \\ \mathrm{CFT} & \mathrm{L} \end{pmatrix} \qquad i = \text{Einstein, log}$$

$$< \text{Einstein} \mid \text{Log} > \neq 0 \quad \Rightarrow$$

$$|S> \equiv \; | \text{Log} > + \alpha | \text{ massless} > \text{ can have either sign!} \;\rightarrow$$

LCFT is non-unitary : non-trivial unitary truncations ?

- away from critical point
- multi-critical points

Lü, Pang, Pope (2011)

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Toy Model

Consider r scalars on a fixed AdS_{D+1} backgound :

$$S = -\frac{1}{2}\int d^{D+1}x\sqrt{g}\sum_{i,j=1}^{r}\left(A_{ij}\partial_\mu\phi_i\partial^\mu\phi_j + B_{ij}\phi_i\phi_j\right)$$

The field equations are given by

$(\Box - m^2)^r\phi_r = 0$ with $r-1$ auxiliary fields $\phi_1\,,\cdots\,,\phi_{r-1}$

$r = 2$: $(\Box - m^2)\phi_2 = \phi_1$ and $(\Box - m^2)\phi_1 = 0$

Boundary Conformal Field Theory

$$\text{rank 3 LCFT} : \quad \langle \mathcal{O}^i \mathcal{O}^j \rangle \sim \begin{pmatrix} 0 & 0 & \mathrm{CFT} \\ 0 & \mathrm{CFT} & \mathrm{L} \\ \mathrm{CFT} & \mathrm{L} & \mathrm{L}^2 \end{pmatrix} \quad i = \text{Einstein, log, log}^2$$

Truncating the $\log^2$ modes leads to

$$\langle \mathcal{O}^i \mathcal{O}^j \rangle \sim \begin{pmatrix} 0 & 0 \\ 0 & \mathrm{CFT} \end{pmatrix} \qquad i = \text{Einstein, log}$$

with a non-negative scalar product !

- Interactions ?

A Gravity Realization ?

Three dimensions : $\mathcal{L} \sim \Lambda + R + R^2 + R\Box R + R_{\mu\nu}\Box R^{\mu\nu}$

Spectrum reduces to one massless and two massive spin two modes

At critical point: $\boxed{\mathcal{G}_{\mu\nu}\big(\mathcal{G}(\mathcal{G}(h))\big) = 0}$

- one tri-critical and three bi-critical points
- positive mass log black holes ?

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Summary

Critical Gravity $\Leftrightarrow$ LCFT

$$\Downarrow$$

Higher-Derivative Critical Gravity $\Leftrightarrow$ Higher-Rank LCFT

- LCFT is in general non-unitary

Open Issues

- are non-trivial unitary truncations possible ?

- extend to most general 4D tri-critical gravity

cp. to Nutma (2012), to appear